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Title: The Monge-Ampère Equation and its Applications

Abstract:

In this talk we will discuss recent researches on the Monge-Ampère equation and its applications. The Monge-Ampère equation

$$\det D^2 u = f$$

is a second order partial differential equation, where D^2u is the Hessian matrix of a function u defined in the Euclidean n-space, and det stands for determinant. The Monge-Ampère operator has some very nice features. It is the product of all eigenvalues of the Hessian matrix, and is the Jacobian determinant of the gradient mapping, and is also invariant under unimodular linear transformations. Therefore the equation found many applications in geometry and physics. In particular it is the prescribing Gauss curvature equation with Gauss curvature K if $f = K(1 + |Du|^2)^{(n+2)/2}$.

The equation is fully nonlinear and is of mixed elliptic-hyperbolic type in general. It is elliptic when u is a convex or concave function. The existence and regularity of elliptic solutions has been well understood through efforts of many researchers. In the last decade striking advances have been made in applications in the reflector design and the optimal transportation, and also in affine geometry. We will also briefly mention some other closely related fully nonlinear equations of Monge-Ampère type with applications in conformal and complex geometry.