



Macquarie University

MIDYEAR EXAMINATION 2006

<i>Unit:</i>	MATH123 – Mathematics 123
<i>Date:</i>	Monday, 26 June 2006 at 9.20 am
<i>Time Allowed:</i>	THREE (3) hours, plus 10 minutes reading time.
<i>Number of questions:</i>	FOURTEEN (14)
<i>Instructions:</i>	All questions may be attempted. The questions are not of equal value. Parts of questions are not of equal value. Use separate books for parts A, B, C and D.
<i>Materials Permitted:</i>	Calculators without text retrieval are permitted, but Graphing Calculators are not permitted. No dictionaries permitted.

7. a) Find expressions for

[6 Marks]

(i) $\int (3x + 4)^5 dx$

(ii) $\int \frac{3}{2x + 1} dx$

- b) Evaluate

(i) $\int_0^1 3e^{2x-1} dx$

(ii) $\int_1^2 \left(x^2 + x + \frac{1}{x} \right) dx$

PART C (Algebra)

(Use a separate book.)

8. a) Given the series: $9 - 6 + 4 - \frac{8}{3} + \frac{16}{9} - \dots$, determine whether a sum to infinity exists. If it does, then find its value.

[8 Marks]

- b) The government, through a subsidy program, distributes \$12,000,000. Assume each individual or agency spends 80% of what they received, and then 80% of this is re-spent by the retailers (say), who then spend 80% of what they received, and so on. How much spending results from the original \$12,000,000? Explain your reasoning.

- c) Use the Binomial Theorem to expand $(2x^2 - 3y)^6$.

9. a) There are 18 starters in the 2006 Macquarie Cup horse race. For the following questions, give your answer as an exact whole number. (Only if you don't have a calculator may you leave it as a product of whole numbers.)

[6 Marks]

- (i) To win a *boxed-trifecta* bet a punter must select the first three horses to finish, in any order.

How many different boxed-trifecta combinations are possible?

- (ii) To win a *superfecta* bet a punter must select the first four place-getters, in the correct order.

How many different superfecta combinations are possible?

10. a) Given matrices $A = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 5 & 2 \\ 2 & 1 & -3 \end{pmatrix}$, $C = \begin{pmatrix} 6 & -4 \\ -9 & 6 \end{pmatrix}$,

[11 Marks]

evaluate each of the following if it exists:

- (i) $2A + 3C$ (ii) CB (iii) BA (iv) A^{-1}
 (v) C^{-1} .

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- b) Express the following system of equations as a matrix equation:

$$2x - 3y = 9 \quad \text{and} \quad 3x + 5y = 4.$$

Solve the matrix equation and use that to solve the system of equations.

PART D (Calculus)

(Use a separate book.)

- 11.** [3 Marks] Sketch the region between the curves $y = x^4$ and $y = x^2$, for $x \geq 0$, and determine the area of the region.
- 12.** [8 Marks] a) Use Newton's method to find an approximate solution of $x^3 + 2x - 1 = 0$, with an initial guess $x_0 = \frac{1}{2}$. Give your answer correct to 4 decimal places.
- b) Estimate the area enclosed by $y = 2x^3$ and the x -axis, for $0 \leq x \leq 2$, using Simpson's rule with 2 subintervals.
- 13.** [7 Marks] At $t = 0$, 10% of all stockbrokers have heard about the impending financial collapse of a large airline. Two hours later, 25% have heard about it. If the proportion of stockbrokers who have heard the rumour is given by $p(t) = \frac{1}{1 + Ce^{-kt}}$, where t is measured in hours,
- Determine the value of C .
 - Determine the value of k (leave your answer as a logarithm).
 - How long before 75% of stockbrokers have heard the rumour? (Hint: you can do this without a calculator if you think about how logarithms work.)
- 14.** [7 Marks] We wish to construct a tank with a square base, rectangular sides and no top. The tank must hold 4m^3 of water. The material from which the tank is to be made costs \$10 per square metre.
- Write an expression for the volume of the tank, in terms of the length of the base (x) and the height (h).
 - Write an expression for the surface area of the tank, in terms of the length of the base (x).
 - Find the value of x which will give the cheapest tank.
 - Write down the dimensions of the tank.
 - Calculate the amount of material needed to construct the tank, and the cost of the material.